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The computation of turbulent flows necessitate a better understanding of turbulence and the development of algorithms and computational tools which are well adapted to the handling of large numbers of data.

Turbulent flows are due to the superposition of a range of small and large eddies which interact and the study of their interaction is an important part of understanding turbulence. An inertial manifold is an exact (quasi-static) interaction law between small and large eddies.

In relation with the concept of approximate inertial manifolds (AIM), Foias–Manley–Temam have shown the existence of a simple finite–dimensional manifold lying close to the attractor. By projecting the Navier–Stokes equations on this manifold we obtain a new numerical algorithm called the Nonlinear Galerkin Method. This algorithm is well–adapted to the large time solution of the Navier–Stokes equations and this has been broadly confirmed by the numerical tests which has been performed during this contract. After further tests and studies, this algorithm will soon be available for industrial implementations.

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Theoretical and Computational Aspects of Turbulence

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Abstract

The computation of turbulent flows necessitate a better understanding of turbulence and the development of algorithms and computational tools which are well adapted to the handling of large numbers of data.

Turbulent flows are due to the superposition of a range of small and large eddies which interact and the study of their interaction is an important part of understanding turbulence. An inertial manifold is an exact (quasi–static) interaction law between small and large eddies.

In relation with the concept of approximate inertial manifolds (AIM), Foias–Manley–Temam have shown the existence of a simple finite–dimensional manifold lying close to the attractor. By projecting the Navier–Stokes equations on this manifold we obtain a new numerical algorithm called the Nonlinear Galerkin Method. This algorithm is well–adapted to the large time solution of the Navier–Stokes equations and this has been broadly confirmed by the numerical tests which has been performed during this contract. After further tests and studies, this algorithm will soon be available for industrial implementations.

For a turbulent flow, the permanent regime is mathematically represented by an attractor that is a compact set in the phase space. Although this set may be highly complicated (fractal), accurate numerical procedures for the solution of the Navier–Stokes equations must produce approximate solutions that are as close as possible from the attractor $\mathscr{A}$. We have to remember also that for turbulent flows, we have to solve the Navier–Stokes equations on large intervals of time, since the flow remains time dependant even if the forces are time independent.

An ideal solution would be to construct an Inertial Manifold for the Navier–Stokes equations and to find approximate solutions that lie on this manifold. However the existence of such a manifold is not yet proved and furthermore it is not clear if this is a computationally feasible avenue.

The approach that we have initiated in this project consists in determining manifolds that are sufficiently close from the attractor without strictly speaking containing it; and to test numerical algorithms based on such manifolds. Such algorithms are called **Nonlinear Galerkin Methods**.

The attractor $\mathscr{A}$ is a complicated set which may be fractal. It seems convenient to approximate it by a smooth manifold which captures (contains) some of the details of the fine structure of $\mathscr{A}$, but not all of them. We define an Approximate Inertial Manifold (AIM) as a smooth manifold which attracts all orbits in a thin neighborhood; in particular this neighborhood must contain $\mathscr{A}$. For the Navier–Stokes equations Foias–Manley–Temam have constructed such a manifold (see [FMT]) and its equation is very simple. Other AIMs have been produced in [T2].

The usual Galerkin method for an evolution equation consists in projecting this equation onto a finite dimensional manifold, namely that spanned by the functions of the basis. A nonlinear Galerkin method consists in projecting the equations to be approximated on a nonlinear manifold; of course we have in view to project the equation on an approximate inertial manifold.

During this project, the nonlinear Galerkin method based on the [FMT] manifold has been numerically implemented and tested, in the case of 2D space periodic flows. A predictor–corrector time discretization scheme was considered and the space discretization was made by collocation–spectral (Fourier series) methods (B. Michaux, C. Rosier, R. Temam). The preliminary numerical results were reported in [T3]. More important numerical tests using the Cray–2 machine were also performed. They show that very slight modifications of existing codes can yield a gain in CPU computing time of 30 to 50%. Also the extension of these algorithms to finite elements, finite differences and wavelets have been initiated [C], [P]. The preliminary results are extremely encouraging.

If we consider the Navier–Stokes equations written in functional form (i.e. by projecting them on the space of divergence free vector functions, we find

$$\frac{du}{dt} + \nu Au + B(u,u) = f\,.$$

The operator A is essentially the Stokes operator associated to the corresponding boundary values condition. It possesses a complete sequence of eigenvectors w_j,

$$Aw_j = \lambda_j w_j,$$
$$0 < \lambda_1 \leq \lambda_2 \leq \dots,\ \lambda_j \to \infty \text{ as } j \to \infty$$

Now $u = u(t)$ is decomposed in the Fourier basis corresponding to the eigenfunctions w_j,

(1) $$u(t) = \sum_{j=1}^{\infty} u_j(t) w_j$$

and we consider the splitting of the sum in (1) at some value m,

$$y_m(t) = \sum_{j=1}^{m} u_j(t)w_j,\ z_m(t) = \sum_{j=m+1}^{\infty} u_j(t)w_j \tag{2}$$
$$u(t) = y_m(t) + z_m(t)\ .$$

We can say that y_m represents the large eddies and z_m represents the small ones. The manifold produced in [FMT] represents an approximate relation between small and large eddies and is:

$$\nu A z_m(t) + Q_m B(y_m(t), y_m(t)) = Q_m f, \tag{3}$$

where Q_m is the projector on the space spanned by $w_{m+1}, \dots$. The construction of the nonlinear Galerkin method based on the approximation (3) of the attractor is explained in details in [T3].

We have also introduced another totally different method of approximating an attractor; see [FT2].

Further work done by B. Michaux and J. Shen (post docs supported in part by this grant) include [S] and [MRS1,2].

This two–year period has been also an initialization period for computing at the Institute for Applied Mathematics and Scientific Computing. The SUNs and the Titan machine acquired with funds from Indiana University and two NSF–SCREMS proposals were delivered in April 1988 and in July 1989. Due to the lack of previous computational environment at I.U. the role of the two post–doctoral researchers, partly supported by AFOSR, (Michaux–Shen) during this initialization period was invaluable.

Although this may not be transparent from our reports and proposals, we are convinced that the algorithms and the methodology presented here and initiated during this contract will play a very important (if not central) role in scientific computing during the *next decade. It is unfortunate that the agency did not decide to pursue its support and be associated with the future developments. However we are extremely grateful for the support already provided.*

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